Session Types Course [Exercise Class 1]

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Notation: we use the symbol $\mathbf{0}$ to denote the process **inact** *(because I'm lazy)*

Definition A ***prefix*** is a process of one of the following forms

$$x[].P \qquad x().P \qquad x[y].P \qquad x(y).P$$

A process P is in ***canonical form*** if $P = (\nu x_1 y_1)\ldots(\nu x_n y_n)(P_1 \mid \cdots \mid P_m)$ with P_i (called ***thread***) a prefix containing no ν-bindings, and x_i or y_i occurring in a P_j for all i and some j in $\{1,\ldots,m\}$.

Exercise 1. Prove that each process P is structurally equivalent to a process P' in canonical form.

Definition Let P and Q be processes. We say that P ***reduces*** to Q if there are $P_1,\ldots,P_n$ such that $P \equiv P_1 \to \cdots \to P_n \equiv Q$ and that P is ***reducible*** if it reduces to a process Q.

Exercise 2. Check if the following processes reduce to $\mathbf{0}$.

1. $x[u].x[].\mathbf{0} \mid y(v).y().\mathbf{0}$
2. $(\nu xy)\big(x[u].x[].\mathbf{0} \mid y(v).y().\mathbf{0}\big)$
3. $(\nu xy)\big(x[u].x(w).x[].\mathbf{0} \mid y(v).y[z].y().\mathbf{0}\big)$
4. $(\nu xy)\big(x[u].x(w).\mathbf{0} \mid y(v).y[z].\mathbf{0}\big)$
5. $(\nu xy)\big(x[u].x[].\mathbf{0} \mid y[v].y[].\mathbf{0}\big)$
6. $(\nu xy)\big(x[u].x(w).x[].\mathbf{0} \mid y(v).y(z).y[].\mathbf{0}\big)$
7. $(\nu xy)\big(x(a).y[b].y().x[].\mathbf{0}\big)$
8. $(\nu xy)\big(x(a).x().\mathbf{0}\big)$
9. $(\nu xy)\big(x[u].x[].\mathbf{0} \mid y(v).y().\mathbf{0} \mid a[b].a().\mathbf{0} \mid c(d).c[].\mathbf{0}\big)$
10. $(\nu x_1 y_1)(\nu x_2 y_2)\big(x_1[a].x_2(b).x_1[].x_2[].\mathbf{0} \mid y_2[c].y_1(d).y_1[].y_2[].\mathbf{0}\big)$

Recall: a process is ***linear*** if each name occurs in at most a thread.

Exercise 3. Let P be a process. Prove that if P is linear, then if $P \to Q_1$ and $P \to Q_2$ with Q_1 and Q_2 irreducible, then $Q_1 \equiv Q_2$.

Definition A process P is ***typable*** if there is a typing derivation of a judgment of the form $\Gamma \vdash P$.

Exercise 4. Which of the processes in Exercise 2 is typable?

Processes

$$
\begin{array}{lcll}
P, Q & := & \mathbf{0} & \text{inact} \\
 & \mid & x[].P & \text{close} \\
 & \mid & x().P & \text{wait} \\
 & \mid & x[y].P & \text{send } (y \text{ through } x) \\
 & \mid & x(y).P & \text{receive } (y \text{ on } x) \\
 & \mid & (\nu xy)P & \text{nu} \\
 & \mid & P \mid Q & \text{parallel}
\end{array}
$$

Structural Equivalence (Processes)

$$
\begin{array}{c}
P \mid \mathbf{0} \equiv P \\
P \mid Q \equiv Q \mid P \\
P \mid (Q \mid R) \equiv (P \mid Q) \mid R \\
(\nu xy)\mathbf{0} \equiv \mathbf{0} \\
(\nu x_1 x_2)(\nu y_1 y_2)P \equiv (\nu y_1 y_2)(\nu x_1 x_2)P \\
((\nu xy)P_1) \mid P_2 \equiv (\nu xy)(P_1 \mid P_2) \\
\alpha.((\nu xy)P) \equiv (\nu xy)(\alpha.P) \\
\boxed{\text{with } x, y \notin \mathsf{fv}(P_2),\ \alpha. \in \{z[].,\, z().,\, z[w].,\, z(w).\}} \\
\text{plus the standard } \alpha\text{-equivalence}
\end{array}
$$

Operational Semantics (Processes)

$$
\begin{array}{rcl}
\textsf{Close:} & (\nu xy)\big(x[].P \mid y().Q\big) \to P \mid Q & \\
\textsf{Com:} & (\nu xy)\big(x[a].P \mid y(b).Q\big) \to (\nu xy)\big(P \mid Q[a/b]\big) & \\
\textsf{Par:} & P \mid Q \to P' \mid Q & \text{if } P \to P' \\
\textsf{Res:} & (\nu xy)P \to (\nu xy)P' & \text{if } P \to P' \\
\textsf{Struct:} & P \to Q & \text{if } P \equiv P' \to Q' \equiv Q
\end{array}
$$

Figure 1: Syntax and semantics for processes

Types

$$
\begin{array}{lcl}
T, U := & := & \textsf{close} \\
 & \mid & \textsf{wait} \\
 & \mid & [T] \blacktriangleleft U \\
 & \mid & (T) \blacktriangleleft U
\end{array}
$$

Duality (for Types)

$$
\frac{\textsf{close} \perp \textsf{wait}}{[T] \blacktriangleleft U \perp (T) \blacktriangleleft V \quad \text{if } U \perp V}
$$

Typing Rules

$$
\textsf{T-Inact}\ \frac{}{\vdash \mathbf{0}}
\qquad
\textsf{T-Par}\ \frac{\Gamma_1 \vdash P \qquad \Gamma_2 \vdash Q}{\Gamma_1, \Gamma_2 \vdash P \mid Q}
\qquad
\textsf{T-Resr}\ \frac{\Gamma, x : T, y : U \vdash P \qquad T \perp U}{\Gamma \vdash (\nu xy)P}
$$

$$
\textsf{T-Close}\ \frac{\Gamma \vdash P}{\Gamma, x : \textsf{close} \vdash x[].P}
\qquad
\textsf{T-Wait}\ \frac{\Gamma \vdash P}{\Gamma, x : \textsf{wait} \vdash x().P}
$$

$$
\textsf{T-Send}\ \frac{\Gamma, x : U, y : T}{\Gamma, x : [T] \blacktriangleleft U \vdash x[y].P}
\qquad
\textsf{T-Recv}\ \frac{\Gamma, x : U}{\Gamma, x : (T) \blacktriangleleft U \vdash x(y).P}
$$

Figure 2: Types