Nested sequents for modal logics and beyond

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In this talk, we will review the different ways *nested sequents* have been used to give cut-free deductive systems for various logics, in particular many that cannot be handled in ordinary (Gentzen) calculi, and other applications as interpolation results, realisation theorems for justification logics, etc.

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