▸ ROMAN KUZNETS, SONIA MARIN, AND LUTZ STRASSBURGER, *Justification logic for modal logics on an intuitionistic base.*
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Justification logic is a family of modal logics generalizing the Logic of Proofs LP, introduced by Artemov in [1]. LP provides an operational view of the same type of provability as modal logic S4. Its language can be seen as a modal language where any occurrence of a formula $\Box A$ would be replaced, using a *justification term*, by $t : A$ with the intended meaning of '*t is a proof of A*'.

Artemov introduced the first intuitionistic version ILP of the Logic of Proofs in [2] to unify the semantics of modalities and lambda-calculus. He shows that ILP is in correspondence with the $\Box$-only fragment of the constructive logic CS4 as defined in [4]. Recently, Marti and Studer [7] supplied ILP with possible worlds semantics akin to the semantics developed by Fitting for the classical Logic of Proofs [6].

Furthermore, in order to obtain an intuitionistic Logic of Proofs that was complete for Heyting arithmetic, Iemhoff and Artemov [3] added to ILP extra axioms that internalize admissible rules of intuitionistic propositional logic. The arithmetical completeness was later shown by Dashkov [5]. Finally, Steren and Bonelli [9] provided an alternative system of terms for ILP based on natural deduction with hypothetical judgements.

What unifies all these versions of intuitionistic justification logics is the exclusive attention to the provability modality. This restriction was quite natural in the classical setting, where $\Diamond$ can simply be viewed as the dual of $\Box$. However, on an intuitionistic base, the De Morgan duality of the $\Box$ and $\Diamond$ modalities is lost and it is possible to treat $\Diamond$ as fully independent [8]. We therefore investigate in this work the kind of terms necessary to represent the operational side of the intuitionistic $\Diamond$ modality.

Namely, building on Artemov's treatment of the $\Box$-only fragment, we propose to add a second type of terms, called *witness terms*, the intuitive understanding of which is based on the view of $\Diamond$ modality as representing consistency (with $\Box$ still read as provability). Thus, a formula $\Diamond A$ is to be realized by '$\mu : A$'. The term μ justifying the consistency of a formula is viewed as an abstract witnessing model for the formula.

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