

- SONIA MARIN, *Intuitionistic modal proof theory: something old, something new*.
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Intuitionistic modal logic, despite more than sixty years of investigation [5, 4, 11], still partly escapes our comprehension. Structural proof theoretic accounts of intuitionistic modal logic have adopted either the paradigm of *labelled deduction* in the form of labelled natural deduction and sequent systems [9], or the one of *unlabelled deduction* in the form of sequent [2] or nested sequent systems [10, 1] (for a survey see [6, Chap. 3]). Both of these approaches are still under active investigation. For instance, Dalmonte *et al* recently proposed a framework for fragments of intuitionistic modal logics [3] using unlabelled sequents that they relate to a new intuitionistic version of *neighbourhood semantics*.

In this talk, we would like to give an overview of the current landscape of intuitionistic modal proof theory and illustrate how these “old and new” approaches can complement each other. We will present a refinement of the labelled approach to what we called a *fully* labelled framework [7]. Taking full advantage of the standard *birelational semantics* of intuitionistic modal logic [8], it allows us to account for intuitionistic modal logic extended with a large class of axioms.

Based on joint work with Mariana Morales and Lutz Straßburger.

[1] RYUTA ARISAKA, ANUPAM DAS AND LUTZ STRASSBURGER, *On Nested Sequents for Constructive Modal Logic*, **Logical Methods in Computer Science**, vol. 1 (2015), no. 3, pp. 1–33.

[2] GAVIN M. BIERMAN AND VALERIA DE PAIVA, *Intuitionistic necessity revisited*, **tech. report**, School of Computer Sciences-University of Birmingham, 1996.

[3] TIZIANO DALMONTE, CHARLES GRELOIS AND NICOLA OLIVETTI, *Intuitionistic Non-Normal Modal Logics: A general framework*, **preprint**, arXiv:1901.09812, 2019.

[4] GISÈLE FISCHER SERVI, *Axiomatizations for some Intuitionistic Modal Logics*, **Rendiconti del Seminario Matematico dell’ Università Politecnica di Torino**, vol. 42 (1984), no. 3, pp. 179–194.

[5] FREDERIC B. FITCH, *Intuitionistic Modal Logic with Quantifiers*, **Portugaliae Mathematica**, vol. 7 (1948), no. 2, pp. 113–118.

[6] SONIA MARIN, *Modal proof theory through a focused telescope*, Ph.D. thesis, Université Paris-Saclay, 2018.

[7] SONIA MARIN, MARIANELA MORALES AND LUTZ STRASSBURGER, *A fully labelled proof system for intuitionistic modal logics*, **preprint**, hal-02390454, 2019.

[8] GORDON D. PLOTKIN AND COLIN P. STIRLING, *A framework for intuitionistic modal logic*, **1st Conference on Theoretical Aspects of Reasoning About Knowledge** (J. Y. Halpern, editor), Morgan Kaufmann, 1986.

[9] ALEX SIMPSON, *The Proof Theory and Semantics of Intuitionistic Modal Logic*, Ph.D. thesis, University of Edinburgh, 1994.

[10] LUTZ STRASSBURGER, *Cut Elimination in Nested Sequents for Intuitionistic Modal Logics*, **16th Conference on Foundations of Software Science and Computation Structures**, (F. Pfenning, editor), Springer, 2013.

[11] DUMINDA WIJESEKERA, *Constructive modal logics I*, **Annals of Pure and Applied Logic**, vol. 50 (1990), no. 3, pp. 271–301.