Modal proof theory through a focused telescope

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and Dale Miller

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Proof theory

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Proof theorists consider proofs as their objects

$\boxed{\text{Object proofs}}$

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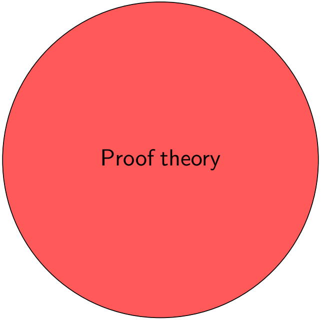
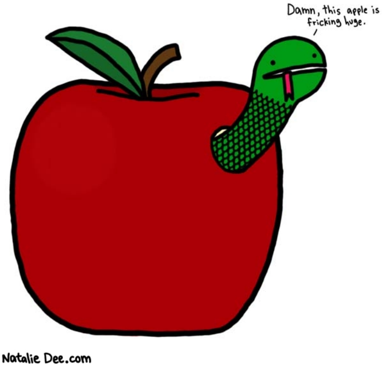
Proof theorists consider proofs as their objects and prove some properties about them.

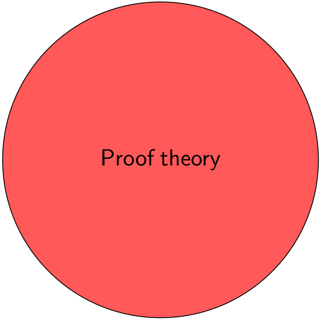
Object proofs

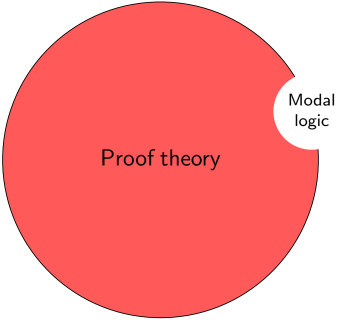
Meta-proofs

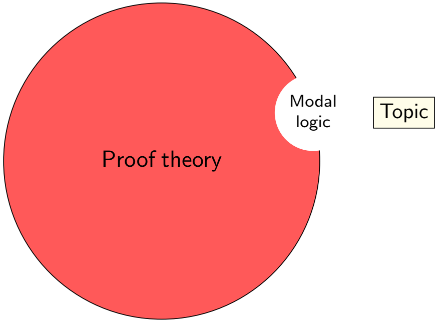

Proof theory


Damn, this apple is fricking huge.
Natalie Dee.com

Proof theory, modal logic and focusing




Modal
logic
Proof theory


Proof theory
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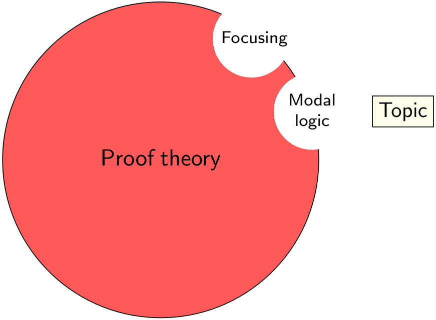
Focusing
Modal
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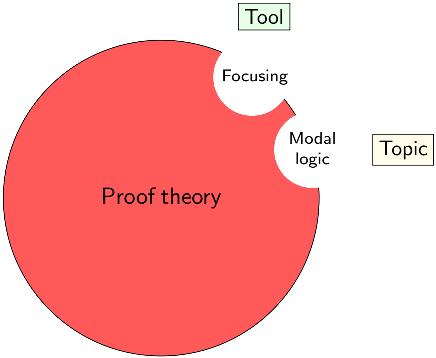
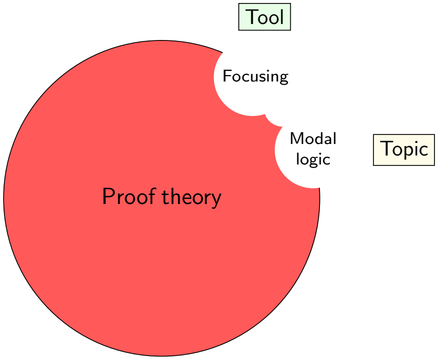
Proof theory, modal logic and focusing



Proof theory, modal logic and focusing



Outline

CARefully Illustrating NEsted sequents

- Logic
- Modal logic
- Modal proof theory

A Unique REarrangement of LIsted ENtities

- Proof search
- Focusing
- Synthetic rules

Actual New Utterances (with Passion And Method)

- Folding
- Unfolding

CARefully Illustrating NEsted sequents

Propositional logic:

$$\left.\begin{array}{lcl} p & : & \textit{it is raining} \\ s & : & \textit{she is sad} \frownie \end{array}\right\}\text{atomic propositions}\left\{\begin{array}{lcl} \bar{p} & : & \text{it is not raining} \\ \bar{s} & : & \textit{she is not sad} \smiley \end{array}\right.$$

Propositional logic:

$$\left.\begin{array}{rcl} p & : & \textit{it is raining} \\ s & : & \textit{she is sad} ☹ \end{array}\right\}\text{atomic propositions}\left\{\begin{array}{rcl} \bar{p} & : & \text{it is } \textcolor{blue}{\text{not}} \text{ raining} \\ \bar{s} & : & \textit{she is } \textcolor{blue}{\textit{not}} \textit{ sad} ☺ \end{array}\right.$$

$$p \supset s \quad : \quad \text{if } \textit{it is raining} \text{ then } \textit{she is sad}$$

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$$\begin{array}{rcl} p \supset s & : & \text{if } \textit{it is raining } \text{then } \textit{she is sad} \\ p \wedge s & : & \textit{it is raining } \text{and } \textit{she is sad} \end{array}$$

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Syntax and basic semantics

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$p \supset s$: if *it is raining* then *she is sad*

$p \wedge s$: *it is raining* and *she is sad*

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Modal logic:

Propositional logic:

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Modal logic:

$\Diamond p$: it is possible that *it is raining*

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$\Diamond p$: it is possible that *it is raining*

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First-order logic:

$$\left.\begin{array}{rcl} l(x) & : & \textit{she loves } x \\ h(x,y) & : & x \textit{ hates } y \end{array}\right\}\text{atomic predicates}$$

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$$\begin{array}{rcl} \forall x. l(x) & : & \textit{she loves all} \text{ things} \\ \exists y. \forall x. h(x, y) \wedge s & : & \text{there exists something that everyone hates and she is sad} \end{array}$$

Possible world semantics

Observed world

$\bar{p}$: it is not raining ☼

s : she is sad ☹

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$V(p) = 0$ $V(s) = 1$

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Possible world semantics

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$\bar{p}$: it is not raining ⛅

s : she is sad ☹

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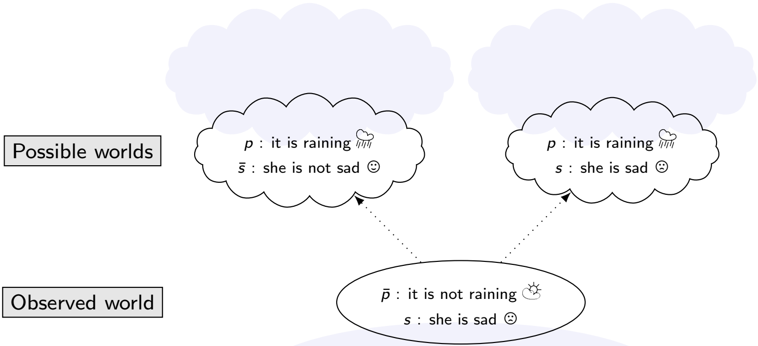
$V(\Box p) = \qquad V(\Diamond s) =$

Possible world semantics



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\begin{array}{cc}
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\end{array}
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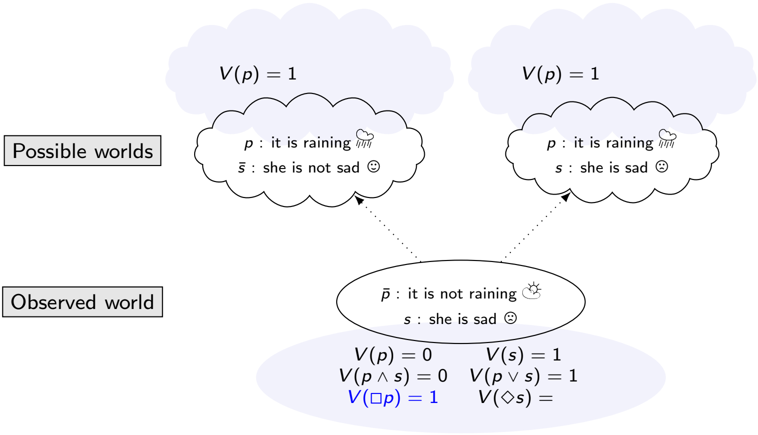
$$w \Vdash \Box A \text{ iff for all } v \text{ such that } w \dashrightarrow v,\ v \Vdash A$$



Possible world semantics

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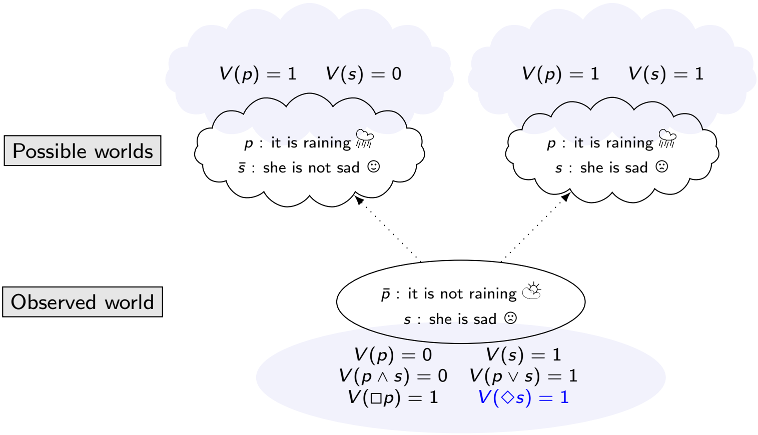

Deduction

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Inference rules:

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- Many different approaches

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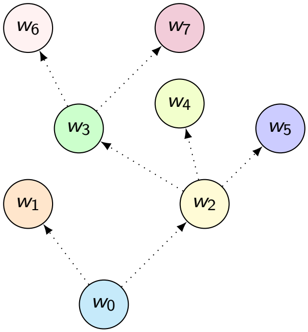
- Many different approaches
- One successful idea!

Syntactical term encoding of the semantical structure

Possible worlds:



[Brünnler, 2009] [Poggiolesi, 2009]

From semantics to syntax

Syntactical term encoding of the semantical structure

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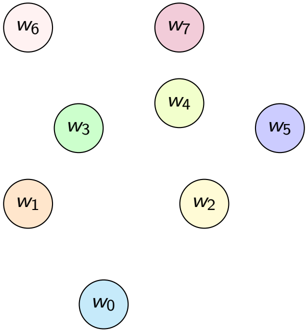

Nested sequents:

$$\left[{}^{0} \qquad \right]$$

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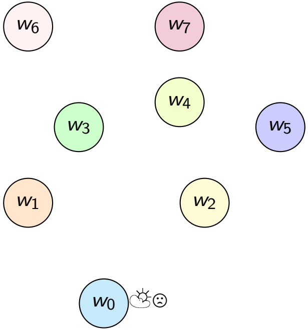

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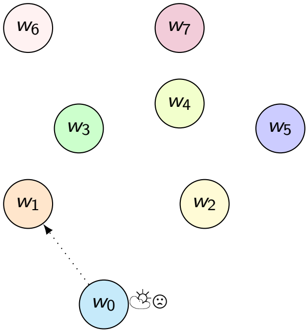

Nested sequents:

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From semantics to syntax

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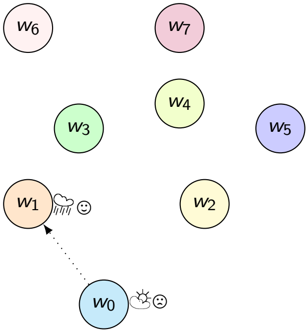

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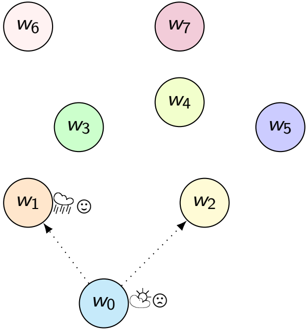

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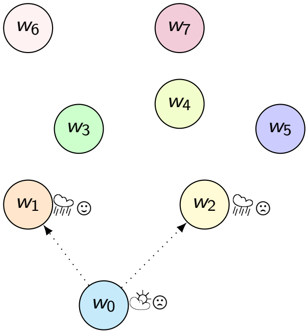

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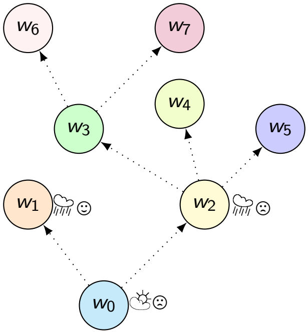

Nested sequents:

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Deduction

Nested sequent system:

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Deduction

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$$

$$
\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y}\ \ldots]}
$$

An example:

$$
\frac{\frac{\frac{\frac{\frac{\overline{} \qquad \overline{}}{}}{}}{}}{}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}
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Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\Box^{\mathsf{n}}\ \frac{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax^n}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\boxed{\Diamond^{\mathsf{n}}\ \frac{[A, \ldots]}{\Diamond A, [\ldots]}}$$

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\boxed{\Diamond^{\mathsf{n}}\ \frac{[A, \ldots]}{\Diamond A, [\ldots]}}$$

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{[b \vee c \qquad]}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\overline{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{\left[^{x} A\right]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{\left[^{y} A, \ldots\right]}{\Diamond A, \left[^{y} \ldots\right]}$$

An example:

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\dfrac{}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$
\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}
$$

$$
\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}
$$

An example:

$$
\boxed{\Diamond^{\mathsf{n}}\ \frac{[A, \ldots]}{\Diamond A, [\ldots]}}
$$

$$
\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\dfrac{\dfrac{\dfrac{}{} \qquad \dfrac{}{}}{}}{}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}
$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{\left[{}^{x} A\right]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{\left[{}^{y} A, \ldots\right]}{\Diamond A, \left[{}^{y}\ \ldots\right]}$$

An example:

$$\boxed{\Diamond^{\mathsf{n}}\ \frac{[A, \ldots]}{\Diamond A, [\ldots]}}$$

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\dfrac{}{\left[a \qquad\qquad\qquad \right]}}{\Diamond a, \left[b \vee c, \bar{a} \wedge \bar{b}\right]}}{\Diamond a, \Diamond(b \vee c), \left[\bar{a} \wedge \bar{b}\right]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$
\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad
\vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad
\vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad
\wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}
$$

$$
\Box^{\mathsf{n}}\ \frac{\left[{}^{x} A\right]}{\Box A} \qquad
\Diamond^{\mathsf{n}}\ \frac{\left[{}^{y} A, \ldots\right]}{\Diamond A, \left[{}^{y} \ldots\right]}
$$

An example:

$$
\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\overline{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}
$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\dfrac{\dfrac{}{} \qquad \dfrac{}{}}{}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[{}^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[{}^{y} A, \ldots]}{\Diamond A, [{}^{y} \ldots]}$$

An example:

$$\boxed{\vee_1^{\mathsf{n}}\ \frac{A}{A \vee B}}$$

$$\Box^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\vee_1^{\mathsf{n}}\ \cfrac{}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \quad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\boxed{\vee_1^{\mathsf{n}}\ \frac{A}{A \vee B}}$$

$$\Box^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\vee_1^{\mathsf{n}}\ \cfrac{\cfrac{\cfrac{}{} \qquad \cfrac{}{}}{b}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Deduction

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \quad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\vee_1^{\mathsf{n}}\ \dfrac{[a,\ b, \bar{a} \wedge \bar{b}]}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Deduction

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\vee_1^{\mathsf{n}}\ \dfrac{\dfrac{\dfrac{}{} \qquad \dfrac{}{}}{[a,\ b, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\boxed{\wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}}$$

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\vee_1^{\mathsf{n}}\ \dfrac{\wedge^{\mathsf{n}}\ \dfrac{\overline{\qquad} \qquad \overline{\qquad}}{[a,\ b, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$
\mathsf{ax}^{\mathsf{n}}\ \frac{}{a,\bar{a}}
\qquad
\vee_1^{\mathsf{n}}\ \frac{A}{A\vee B}
\qquad
\vee_2^{\mathsf{n}}\ \frac{B}{A\vee B}
\qquad
\wedge^{\mathsf{n}}\ \frac{A \qquad B}{A\wedge B}
$$

$$
\Box^{\mathsf{n}}\ \frac{\left[{}^{x} A\right]}{\Box A}
\qquad
\Diamond^{\mathsf{n}}\ \frac{\left[{}^{y} A, \ldots\right]}{\Diamond A, \left[{}^{y} \ldots\right]}
$$

An example:

$$
\boxed{\wedge^{\mathsf{n}}\ \frac{A \qquad B}{A\wedge B}}
$$

$$
\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\vee_1^{\mathsf{n}}\ \dfrac{\wedge^{\mathsf{n}}\ \dfrac{\dfrac{}{\bar{a}} \qquad \dfrac{}{\bar{b}}}{[a,\ b,\ \bar{a}\wedge\bar{b}]}}{[a, b\vee c, \bar{a}\wedge\bar{b}]}}{\Diamond a, [b\vee c, \bar{a}\wedge\bar{b}]}}{\Diamond a, \Diamond(b\vee c), [\bar{a}\wedge\bar{b}]}}{\Diamond a, \Diamond(b\vee c), \Box(\bar{a}\wedge\bar{b})}
$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\Box^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\vee_1^{\mathsf{n}}\ \cfrac{\wedge^{\mathsf{n}}\ \cfrac{\cfrac{}{[a, b, \bar{a}]} \qquad \cfrac{}{[a, b, \bar{b}]}}{[a, b, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\vee_1^{\mathsf{n}}\ \dfrac{\wedge^{\mathsf{n}}\ \dfrac{\overline{[a, b, \bar{a}]} \qquad \overline{[a, b, \bar{b}]}}{[a, b, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond (b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond (b \vee c), \Box (\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$
\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}}
\qquad
\vee_1^{\mathsf{n}}\ \frac{A}{A \vee B}
\qquad
\vee_2^{\mathsf{n}}\ \frac{B}{A \vee B}
\qquad
\wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}
$$

$$
\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A}
\qquad
\Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}
$$

An example:

$$
\boxed{\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}}}
$$

$$
\Box^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\vee_1^{\mathsf{n}}\ \cfrac{\wedge^{\mathsf{n}}\ \cfrac{\mathsf{ax}^{\mathsf{n}}\ \cfrac{}{[a, b, \bar{a}]} \qquad \cfrac{}{[a, b, \bar{b}]}}{[a, b, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}
$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a, \bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A \wedge B}$$

$$\Box^{\mathsf{n}}\ \frac{[^{x} A]}{\Box A} \qquad \Diamond^{\mathsf{n}}\ \frac{[^{y} A, \ldots]}{\Diamond A, [^{y} \ldots]}$$

An example:

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\vee_1^{\mathsf{n}}\ \dfrac{\wedge^{\mathsf{n}}\ \dfrac{\mathsf{ax}^{\mathsf{n}}\ \dfrac{}{[a, b, \bar{a}]} \qquad \dfrac{}{[a, b, \bar{b}]}}{[a, b, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

Nested sequent system:

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{a,\bar{a}} \qquad \vee_1^{\mathsf{n}}\ \frac{A}{A\vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A\vee B} \qquad \wedge^{\mathsf{n}}\ \frac{A \qquad B}{A\wedge B}$$

$$\square^{\mathsf{n}}\ \frac{[{}^{x}A]}{\square A} \qquad \Diamond^{\mathsf{n}}\ \frac{[{}^{y}A,\ldots]}{\Diamond A,[{}^{y}\ldots]}$$

An example:

$$\boxed{\mathsf{ax}^{\mathsf{n}}\ \frac{}{a,\bar{a}}}$$

$$\square^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\Diamond^{\mathsf{n}}\ \cfrac{\vee_1^{\mathsf{n}}\ \cfrac{\wedge^{\mathsf{n}}\ \cfrac{\mathsf{ax}^{\mathsf{n}}\ \cfrac{}{[a,b,\bar{a}]} \qquad \mathsf{ax}^{\mathsf{n}}\ \cfrac{}{[a,b,\bar{b}]}}{[a,b,\bar{a}\wedge\bar{b}]}}{[a,b\vee c,\bar{a}\wedge\bar{b}]}}{\Diamond a,[b\vee c,\bar{a}\wedge\bar{b}]}}{\Diamond a,\Diamond(b\vee c),[\bar{a}\wedge\bar{b}]}}{\Diamond a,\Diamond(b\vee c),\square(\bar{a}\wedge\bar{b})}$$

Deduction

Nested sequent system: $\Gamma\{\ \} ::= \{\ \} \mid A, \Gamma\{\ \} \mid \Gamma, [\Gamma\{\ \}]$

$$\mathsf{ax}^{\mathsf{n}}\ \frac{}{\Gamma\{a, \bar{a}\}} \qquad \vee_1^{\mathsf{n}}\ \frac{\Gamma\{A\}}{\Gamma\{A \vee B\}} \qquad \vee_2^{\mathsf{n}}\ \frac{\Gamma\{B\}}{\Gamma\{A \vee B\}} \qquad \wedge^{\mathsf{n}}\ \frac{\Gamma\{A\} \quad \Gamma\{B\}}{\Gamma\{A \wedge B\}}$$

$$\Box^{\mathsf{n}}\ \frac{\Gamma\{[A]\}}{\Gamma\{\Box A\}} \qquad \Diamond^{\mathsf{n}}\ \frac{\Gamma_1\{[A, \Gamma_2]\}}{\Gamma_1\{\Diamond A, [\Gamma_2]\}}$$

An example:

$$\Box^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\Diamond^{\mathsf{n}}\ \dfrac{\vee_1^{\mathsf{n}}\ \dfrac{\wedge^{\mathsf{n}}\ \dfrac{\mathsf{ax}^{\mathsf{n}}\ \dfrac{}{[a, b, \bar{a}]} \quad \mathsf{ax}^{\mathsf{n}}\ \dfrac{}{[a, b, \bar{b}]}}{[a, b, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}$$

A Unique REarrangement of LIsted ENtities

Proving is choosing

Choice of focus:

$$\Diamond^{\mathsf{n}}\ \frac{\Diamond a,\ \left[b \vee c,\ \bar{a} \wedge \bar{b}\right]}{\Diamond a,\ \Diamond(b \vee c),\ \left[\bar{a} \wedge \bar{b}\right]} \qquad \Diamond^{\mathsf{n}}\ \frac{\Diamond(b \vee c),\ \left[a,\ \bar{a} \wedge \bar{b}\right]}{\Diamond a,\ \Diamond(b \vee c),\ \left[\bar{a} \wedge \bar{b}\right]} \qquad \wedge^{\mathsf{n}}\ \frac{\Diamond a,\ \Diamond(b \vee c),\ [\bar{a}] \quad \Diamond a,\ \Diamond(b \vee c),\ \left[\bar{b}\right]}{\Diamond a,\ \Diamond(b \vee c),\ \left[\bar{a} \wedge \bar{b}\right]}$$

Choice of focus:

$$\Diamond^{\mathsf{n}}\ \frac{\Diamond a,\ [b \vee c, \bar{a} \wedge \bar{b}]}{\Diamond a,\ \Diamond(b \vee c),\ [\bar{a} \wedge \bar{b}]} \qquad \Diamond^{\mathsf{n}}\ \frac{\Diamond(b \vee c),\ [a, \bar{a} \wedge \bar{b}]}{\Diamond a,\ \Diamond(b \vee c),\ [\bar{a} \wedge \bar{b}]} \qquad \wedge^{\mathsf{n}}\ \frac{\Diamond a, \Diamond(b \vee c), [\bar{a}] \quad \Diamond a, \Diamond(b \vee c), [\bar{b}]}{\Diamond a,\ \Diamond(b \vee c),\ [\bar{a} \wedge \bar{b}]}$$

Choice of rule:

$$\vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B}$$

Choice of focus:

$$\Diamond^{\mathsf{n}}\ \frac{\Diamond a,\ \left[b \vee c,\ \bar{a} \wedge \bar{b}\right]}{\Diamond a,\ \Diamond(b \vee c),\ \left[\bar{a} \wedge \bar{b}\right]} \qquad \Diamond^{\mathsf{n}}\ \frac{\Diamond(b \vee c),\ \left[a,\ \bar{a} \wedge \bar{b}\right]}{\Diamond a,\ \Diamond(b \vee c),\ \left[\bar{a} \wedge \bar{b}\right]} \qquad \wedge^{\mathsf{n}}\ \frac{\Diamond a,\ \Diamond(b \vee c),\ [\bar{a}] \quad \Diamond a,\ \Diamond(b \vee c),\ \left[\bar{b}\right]}{\Diamond a,\ \Diamond(b \vee c),\ \left[\bar{a} \wedge \bar{b}\right]}$$

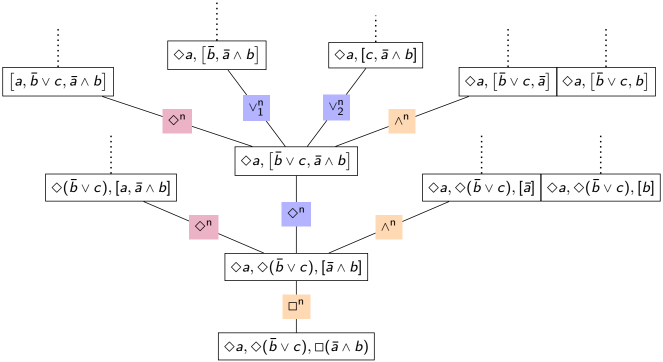
Choice of rule:

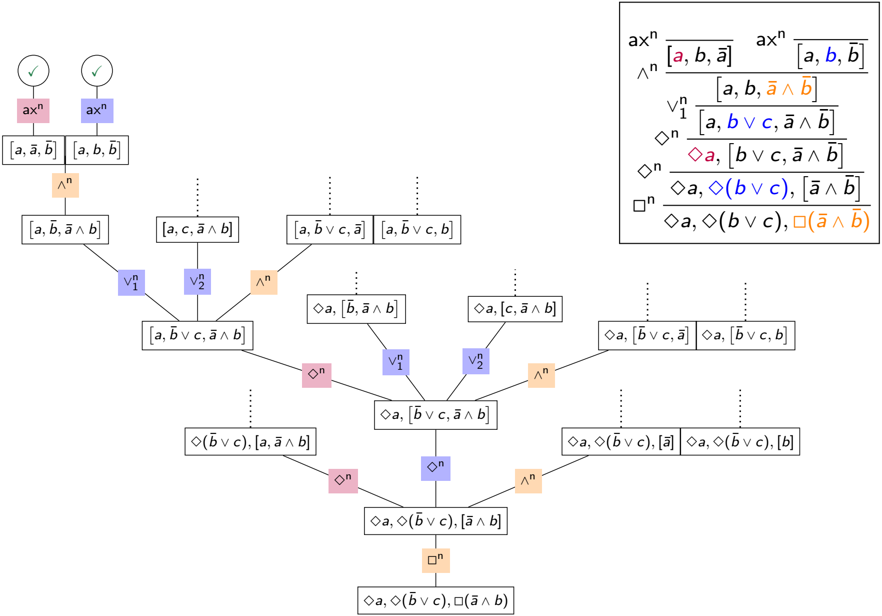
$$\vee_1^{\mathsf{n}}\ \frac{A}{A \vee B} \qquad \vee_2^{\mathsf{n}}\ \frac{B}{A \vee B}$$

Choice of context:

$$\Diamond_1^{\mathsf{n}}\ \frac{\left[{}^1 A, \ldots\right], \left[{}^2 \ldots\right]}{\Diamond A, \left[{}^1 \ldots\right], \left[{}^2 \ldots\right]} \qquad \Diamond_2^{\mathsf{n}}\ \frac{\left[{}^1 \ldots\right], \left[{}^2 A, \ldots\right]}{\Diamond A, \left[{}^1 \ldots\right], \left[{}^2 \ldots\right]}$$

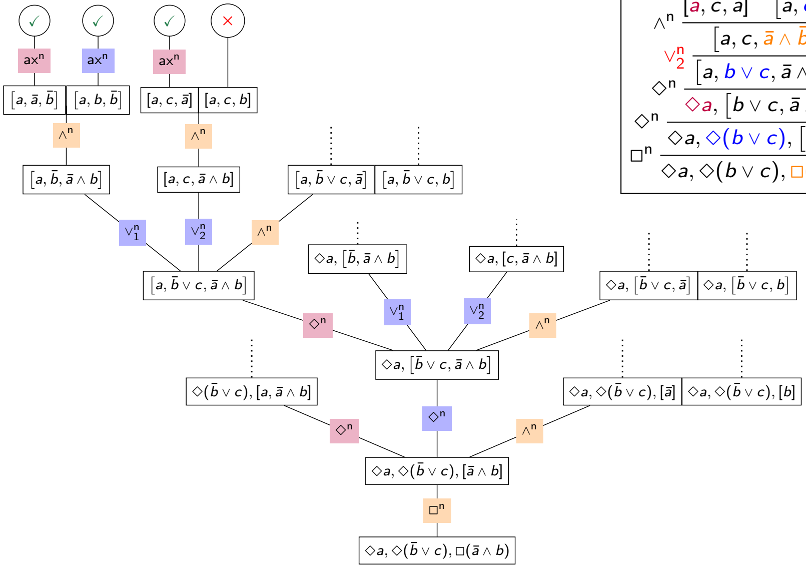
Proof search space



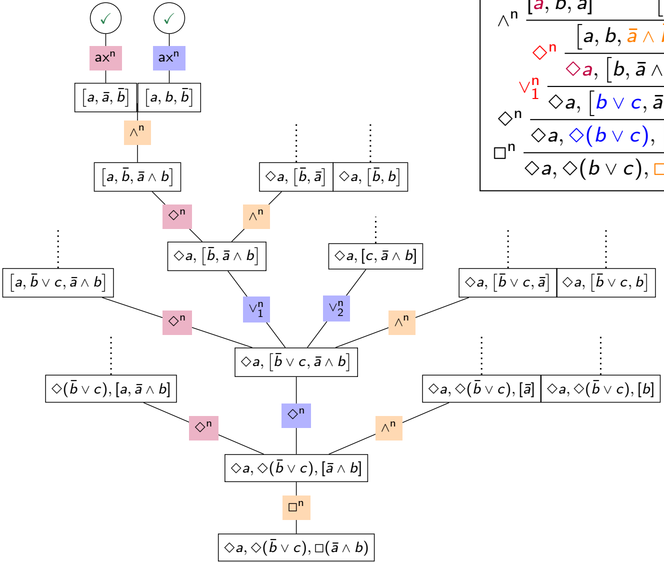


ax^n
ax^n
[a, ā, b̄]
[a, b, b̄]
∧^n
[a, b̄, ā ∧ b]
[a, c, ā ∧ b]
[a, b̄ ∨ c, ā]
[a, b̄ ∨ c, b]
∨^n_1
∨^n_2
∧^n
[a, b̄ ∨ c, ā ∧ b]
◇a, [b̄, ā ∧ b]
◇a, [c, ā ∧ b]
◇a, [b̄ ∨ c, ā]
◇a, [b̄ ∨ c, b]
◇^n
∨^n_1
∨^n_2
∧^n
◇a, [b̄ ∨ c, ā ∧ b]
◇(b̄ ∨ c), [a, ā ∧ b]
◇a, ◇(b̄ ∨ c), [ā]
◇a, ◇(b̄ ∨ c), [b]
◇^n
◇^n
∧^n
◇a, ◇(b̄ ∨ c), [ā ∧ b]
□^n
◇a, ◇(b̄ ∨ c), □(ā ∧ b)
ax^n [a, b, ā]
ax^n [a, b, b̄]
∧^n [a, b, ā ∧ b̄]
∨^n_1 [a, b ∨ c, ā ∧ b̄]
◇^n ◇a, [b ∨ c, ā ∧ b̄]
◇^n ◇a, ◇(b ∨ c), [ā ∧ b̄]
□^n ◇a, ◇(b ∨ c), □(ā ∧ b̄)

Proof search space



$$
\square^n\,\dfrac{\Diamond^n\,\dfrac{\Diamond^n\,\dfrac{\vee_2^n\,\dfrac{\wedge^n\,\dfrac{\mathrm{ax}^n\,\dfrac{}{[a, c, \bar{a}]} \qquad \dfrac{\times}{[a, c, \bar{b}]}}{[a, c, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \square(\bar{a} \wedge \bar{b})}
$$



$$\square^n\ \dfrac{\Diamond^n\ \dfrac{\vee_1^n\ \dfrac{\Diamond^n\ \dfrac{\wedge^n\ \dfrac{\mathrm{ax}^n\ \dfrac{}{[a, b, \bar a]} \qquad \mathrm{ax}^n\ \dfrac{}{[a, b, \bar b]}}{[a, b, \bar a \wedge \bar b]}}{\Diamond a, [b, \bar a \wedge \bar b]}}{\Diamond a, [b \vee c, \bar a \wedge \bar b]}}{\Diamond a, \Diamond(b \vee c), [\bar a \wedge \bar b]}}{\Diamond a, \Diamond(b \vee c), \square(\bar a \wedge \bar b)}$$

Hocus Focus

Focusing provides a way to restrict the proof search space.

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$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{[a, b, \bar{a}]}\,\mathrm{ax}^{\mathrm{n}} \quad \dfrac{}{[a, b, \bar{b}]}\,\mathrm{ax}^{\mathrm{n}}}{[a, b, \bar{a} \wedge \bar{b}]}\,\wedge^{\mathrm{n}}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}\,\vee_1^{\mathrm{n}}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}\,\Diamond^{\mathrm{n}}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}\,\Diamond^{\mathrm{n}}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}\,\Box^{\mathrm{n}}
$$

$\boxed{\text{Unfocused}}$

Focusing provides a way to restrict the proof search space.

- Maximal chaining of the decomposition.

$$
\Box^{n}\,\dfrac{\Diamond^{n}\,\dfrac{\Diamond^{n}\,\dfrac{\vee_1^{n}\,\dfrac{\wedge^{n}\,\dfrac{\mathsf{ax}^{n}\,\dfrac{}{[a, b, \bar{a}]} \qquad \mathsf{ax}^{n}\,\dfrac{}{[a, b, \bar{b}]}}{[a, b, \bar{a} \wedge \bar{b}]}}{[a, b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, [b \vee c, \bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}
\quad\longrightarrow\quad
\Box^{n}\,\dfrac{\wedge^{n}\,\dfrac{\Diamond^{n}\,\dfrac{\mathsf{ax}^{n}\,\dfrac{}{\Diamond(b \vee c), [a, \bar{a}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a}]} \qquad \Diamond^{n}\,\dfrac{\vee^{n}\,\dfrac{\mathsf{ax}^{n}\,\dfrac{}{\Diamond a, [b, \bar{b}]}}{\Diamond a, [b \vee c, \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \Box(\bar{a} \wedge \bar{b})}
$$

Unfocused | Focused

Polarity and focusing

Polarities:

$$\begin{array}{lcl} \text{positive formulas} & : & P ::= \quad a \mid P \vee P \mid \Diamond P \mid {\downarrow} N \\ \text{negative formulas} & : & N ::= \quad \bar{a} \mid N \wedge N \mid \Box N \mid {\uparrow} P \end{array}$$

[Andreoli, 1990] [Laurent, 2004]

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negative formulas : $N ::= \quad \bar{a} \mid N \wedge N \mid \Box N \mid {\uparrow} P$

Focusing:

Inversion: In $\begin{array}{c} \pi \Vert \\ \Gamma\{N\} \end{array}$ the last rule is negative

[Andreoli, 1990] [Laurent, 2004]

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Completeness of focusing: If a formula is provable then it has a focused proof.

[Andreoli, 1990] [Laurent, 2004]

Control over choices in focused proof is known to improve proof search,

$$
\square^{\mathsf{n}}\,\dfrac{\wedge^{\mathsf{n}}\,\dfrac{\Diamond^{\mathsf{n}}\,\dfrac{\mathsf{ax}^{\mathsf{n}}\,\dfrac{}{\Diamond(b \vee c), [a, \bar{a}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a}]} \qquad \Diamond^{\mathsf{n}}\,\dfrac{\vee^{\mathsf{n}}\,\dfrac{\mathsf{ax}^{\mathsf{n}}\,\dfrac{}{\Diamond a, [b, \bar{b}]}}{\Diamond a, [b \vee c, \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}}{\Diamond a, \Diamond(b \vee c), \square(\bar{a} \wedge \bar{b})}
$$

$\boxed{\text{Focused}}$

Control over choices in focused proof is known to improve proof search, but also allows for a compact synthetic representation.

$$
\mathrm{\square^n}\;\dfrac{
\mathrm{\wedge^n}\;\dfrac{
\mathrm{\Diamond^n}\;\dfrac{\mathrm{ax^n}\;\dfrac{}{\Diamond(b \vee c), [a, \bar{a}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a}]}
\qquad
\mathrm{\Diamond^n}\;\dfrac{\mathrm{\vee^n}\;\dfrac{\mathrm{ax^n}\;\dfrac{}{\Diamond a, [b, \bar{b}]}}{\Diamond a, [b \vee c, \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{b}]}
}{\Diamond a, \Diamond(b \vee c), [\bar{a} \wedge \bar{b}]}
}{\Diamond a, \Diamond(b \vee c), \square(\bar{a} \wedge \bar{b})}
\quad \longrightarrow \quad
\mathrm{neg}\;\dfrac{
\mathrm{pos}\;\dfrac{\mathrm{ax^n}\;\dfrac{}{\Diamond(b \vee c), [a, \bar{a}]}}{\Diamond a, \Diamond(b \vee c), [\bar{a}]}
\qquad
\mathrm{pos}\;\dfrac{\mathrm{ax^n}\;\dfrac{}{\Diamond a, [b, \bar{b}]}}{\Diamond a, \Diamond(b \vee c), [\bar{b}]}
}{\Diamond a, \Diamond(b \vee c), \square(\bar{a} \wedge \bar{b})}
$$

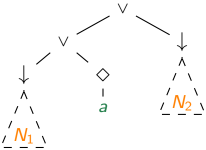
Focused

Synthetic

Synthetic connectives

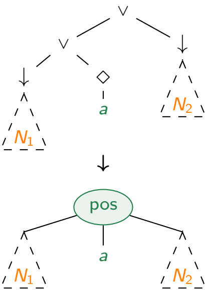
$$
\vee_1^{\mathsf{n}}\ \dfrac{\vee_2^{\mathsf{n}}\ \dfrac{\Diamond\ \dfrac{[a, \ldots]}{\Diamond a, [\ldots]}}{N_1 \vee \Diamond a, [\ldots]}}{(N_1 \vee \Diamond a) \vee N_2, [\ldots]}
$$

$$\vee_1^{\mathsf{n}}\ \dfrac{\vee_2^{\mathsf{n}}\ \dfrac{\Diamond\ \dfrac{[a, \ldots]}{\Diamond a, [\ldots]}}{N_1 \vee \Diamond a, [\ldots]}}{(N_1 \vee \Diamond a) \vee N_2, [\ldots]}$$



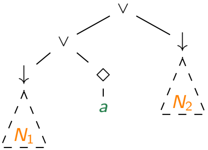
Synthetic connectives

$$\vee_1^{\mathsf{n}}\ \cfrac{\vee_2^{\mathsf{n}}\ \cfrac{\Diamond\ \cfrac{[a, \ldots]}{\Diamond a, [\ldots]}}{N_1 \vee \Diamond a, [\ldots]}}{(N_1 \vee \Diamond a) \vee N_2, [\ldots]}$$

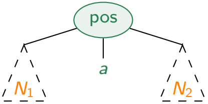


$$\mathsf{pos}\ \frac{[a, \ldots]}{(N_1 \vee \Diamond a) \vee N_2, [\ldots]}$$

$$\vee_1^{n}\,\cfrac{\vee_2^{n}\,\cfrac{\Diamond\,\cfrac{[a,\ldots]}{\Diamond a,[\ldots]}}{N_1 \vee \Diamond a,[\ldots]}}{(N_1 \vee \Diamond a) \vee N_2,[\ldots]}$$

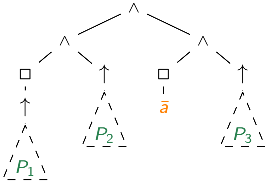


$$\downarrow$$



$$\text{pos}\,\cfrac{[a,\ldots]}{(N_1 \vee \Diamond a) \vee N_2,[\ldots]}$$

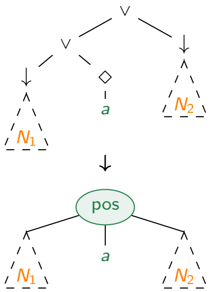
$$\wedge^{n}\,\cfrac{\wedge^{n}\,\cfrac{\Box^{n}\,\cfrac{[P_1]}{\Box P_1} \quad P_2}{\Box P_1 \wedge P_2} \qquad {}^{n}\cfrac{\Box^{n}\,\cfrac{[\bar{a}]}{\Box \bar{a}} \quad P_3}{\Box \bar{a} \wedge P_3}}{(\Box P_1 \wedge P_2) \wedge (\Box \bar{a} \wedge P_3)}$$

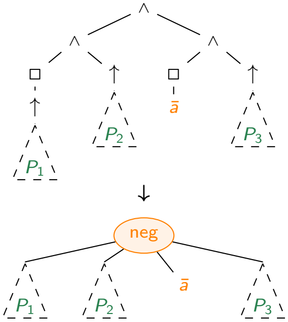


Synthetic connectives

$$\vee_1^{n}\;\dfrac{\vee_2^{n}\;\dfrac{\Diamond\;\dfrac{[a, \ldots]}{\Diamond a, [\ldots]}}{N_1 \vee \Diamond a, [\ldots]}}{(N_1 \vee \Diamond a) \vee N_2, [\ldots]}$$

$$\wedge^{n}\;\dfrac{\wedge^{n}\;\dfrac{\Box^{n}\;\dfrac{[P_1]}{\Box P_1} \quad P_2}{\Box P_1 \wedge P_2} \qquad {}^{n}\dfrac{\Box^{n}\;\dfrac{[\bar{a}]}{\Box \bar{a}} \quad P_3}{\Box \bar{a} \wedge P_3}}{(\Box P_1 \wedge P_2) \wedge (\Box \bar{a} \wedge P_3)}$$





$$\text{pos}\;\dfrac{[a, \ldots]}{(N_1 \vee \Diamond a) \vee N_2, [\ldots]}$$

$$\text{neg}\;\dfrac{[P_1] \quad P_2 \quad [\bar{a}] \quad P_3}{(\Box P_1 \wedge P_2) \wedge (\Box \bar{a} \wedge P_3)}$$

Actual New Utterances (with Passion And Method)

Contributions:

1. Design a system that generates only focused proofs

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   - Reduction of the proof search space

Folding

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Folding

Contributions:

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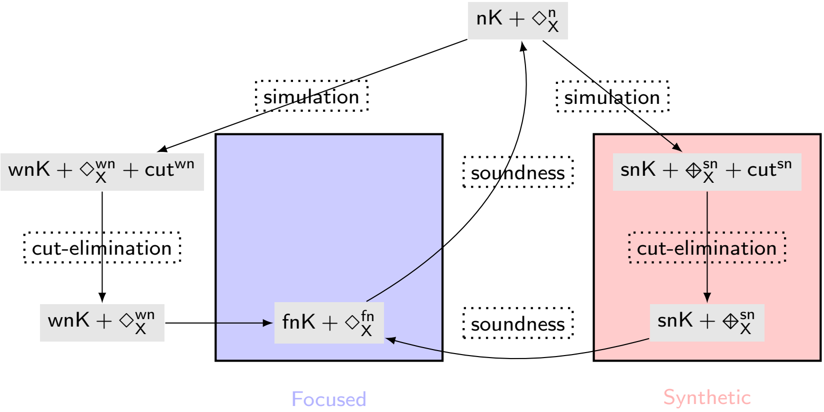
   - Reduction of the proof search space
2. Design a system that generates only synthetic proofs
   - Simple and elegant proof of completeness


$nK + \Diamond_X^n$
simulation
simulation
$wnK + \Diamond_X^{wn} + cut^{wn}$
soundness
$snK + \oplus_X^{sn} + cut^{sn}$
cut-elimination
cut-elimination
$wnK + \Diamond_X^{wn}$
$fnK + \Diamond_X^{fn}$
soundness
$snK + \oplus_X^{sn}$
Focused
Synthetic

Contributions:

1. Design a focused labelled framework that can emulate many standard proof systems for modal logic

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   - checking, (sharing, and reconstructing) proofs

Emulating a nested rule

proof in S $\longleftrightarrow$ proof in LMF_*

$$\Diamond^{\mathsf{n}}_{\mathsf{Rb}}\,\frac{\Lambda_1\{{}^{x}[{}^{y}\Lambda_2], A\}}{\Lambda_1\{{}^{x}[{}^{y}\Lambda_2, \Diamond A]\}}$$

proof in S $\longleftrightarrow$ proof in LMF_* $\longleftrightarrow$ proof in $\mathsf{LKF}^{\mathsf{a}}$

$$\diamond^{\mathsf{n}}_{\mathsf{Rb}} \frac{\Lambda_1\{{}^{x}[{}^{y}\Lambda_2], A\}}{\Lambda_1\{{}^{x}[{}^{y}\Lambda_2, \diamond A]\}}$$

$$
\begin{tikzcd}
\text{proof in S} \arrow[r, leftrightarrow] \arrow[rr, bend left] & \text{proof in LMF}_{*} \arrow[r, leftrightarrow] & \text{proof in LKF}^{\mathsf{a}}
\end{tikzcd}
$$

$$
\Diamond^{\mathsf{n}}_{\mathsf{Rb}}\ \dfrac{\Lambda_1\{{}^{x}[{}^{y}\Lambda_2], A\}}{\Lambda_1\{{}^{x}[{}^{y}\Lambda_2, \Diamond A]\}} \qquad \rightarrow
$$

$$
\downarrow_{\mathsf{L}},\uparrow_{\mathsf{R}}\ \dfrac{
\forall_{\mathsf{L}},\exists_{\mathsf{R}}\ \dfrac{
\overset{+}{\curlywedge}_{\mathsf{R}}\ \dfrac{
\supset_{\mathsf{L}}\ \dfrac{
\mathsf{ax_R}\ \dfrac{}{\phi, r(x,y), \mathcal{L}, \psi, \langle r(x,y)\rangle}
\qquad
\uparrow_{\mathsf{L}}\ \dfrac{\mathsf{ax_R}\ \dfrac{}{r(y,x), \phi, r(x,y), \mathcal{L}, \psi, \langle r(y,x)\rangle}}{\langle\uparrow r(y,x)\rangle, \phi, r(x,y), \mathcal{L}, \psi, \langle r(y,x)\rangle}
}{\langle r(x,y)\supset\uparrow r(y,x)\rangle, \phi, r(x,y), \mathcal{L}, \psi, \langle r(y,x)\rangle}
\qquad
\downarrow_{\mathsf{R}}\ \dfrac{r(x,y), \mathcal{L}, \lceil A\rceil_x}{r(x,y), \mathcal{L}, \psi, \langle\downarrow\lceil A\rceil_x\rangle}
}{\langle r(x,y)\supset\uparrow r(y,x)\rangle, \phi, r(x,y), \mathcal{L}, \psi, \langle r(y,x)\overset{+}{\curlywedge}\downarrow\lceil A\rceil_x\rangle}
}{\langle\forall uv.(r(u,v))\supset\uparrow r(v,u))\rangle, \phi, r(x,y), \mathcal{L}, \psi, \langle\exists z.(r(y,z)\overset{+}{\curlywedge}\downarrow\lceil A\rceil_z)\rangle}
}{\phi = \downarrow\forall uv.(r(u,v)\supset\uparrow r(v,u)), r(x,y), \mathcal{L}, \psi = \uparrow\exists z.(r(y,z)\overset{+}{\curlywedge}\downarrow\lceil A\rceil_z)}
$$

Conclusion

Applications of the focusing toolset to modal proof theory:

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- Folding: multifocusing and identity of proofs
- Unfolding : Emulation of proof systems for other logics / in different formalisms